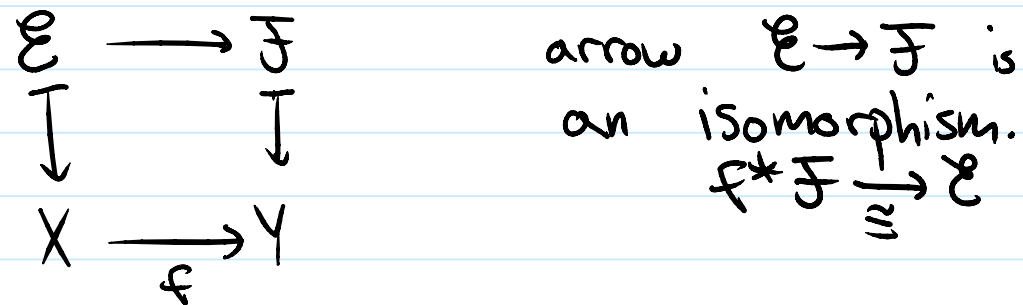
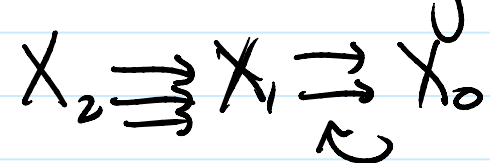


**prop:** A for a morphism  $X_0 \rightarrow Y_0$ ,  $\exists$  pullback functor  $QCoh(Y_0) \rightarrow QCoh(X_0)$ , which is an equivalence if  $f$  is morita, and natural trans. gives iso. of functors

The cleanest way to think of this is given a diagram of schemes, what is a Quasi-coherent sheaf on that diagram (Cartesian)



Claim: A quasicohereant sheaf is the same as a Cartesian quasicohereant sheaf on the diagram

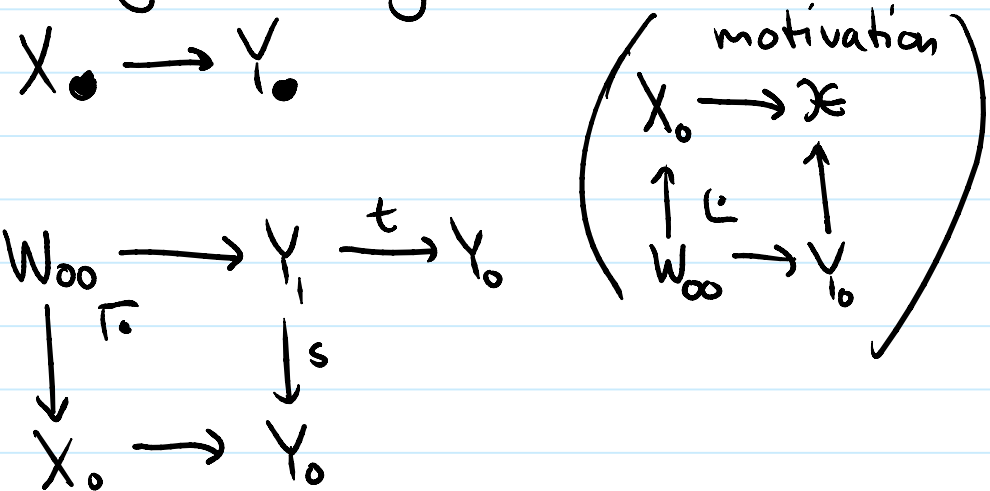


Proof of prop:

Step 1) Banal groupoid for  $f \circ f \circ f$  map

Step 1) Banal groupoid for fppf map  
 $\hookrightarrow$  can reduce to case of affine fppf maps, statement about ring maps

2) for general groupoids, look at



3) Use the fact that

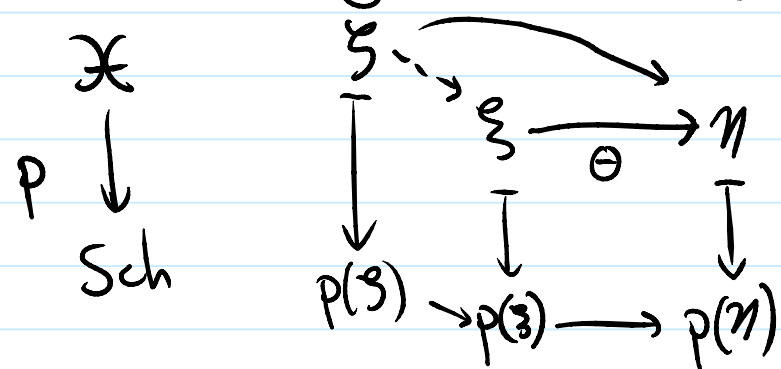
$$\mathrm{QCoh}(X_{\bullet}) \xrightarrow{\cong} \mathrm{QCoh}(W_{\bullet\bullet}) \xleftarrow{\cong} \mathrm{QCoh}(Y_{\bullet})$$



Thus we think of  $\mathrm{QCoh}(X_{\bullet})$  as indep. of presentation.

Already, we secretly used the notion of fibered categories. Definition: existence

Already, we secretly used the notion of fibered categories: definition: existence of Cartesian arrows

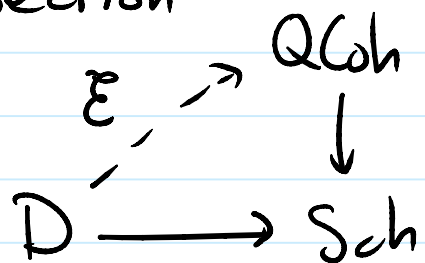


We saw the example of  $QCoh = \{(X, \mathcal{E})\}$

Think of this as a functor from schemes to categories

$$QCoh(U) := p^{-1}(U) \subset QCoh$$

Given a diagram of schemes, a quasi-coherent sheaf on that diagram is a Cartesian section



This leads to the correct notion of a stack (modeled after the stack of q-coh. sheaves).

Def: A stack is a category fibered in groupoids such that  $\forall$  banal groupoids associated to an etale cover  $U \rightarrow X$  of schemes,

$\mathcal{F}(X) \rightarrow \mathcal{F}(U)$   
is an equivalence,

(Vistoli)

#### 4.1.3. Fibered categories with descent.

DEFINITION 4.6. Let  $\mathcal{F} \rightarrow \mathcal{C}$  be a fibered category on a site  $\mathcal{C}$ .

- (i)  $\mathcal{F}$  is a *prestack* over  $\mathcal{C}$  if for each covering  $\{U_i \rightarrow U\}$  in  $\mathcal{C}$ , the functor  $\mathcal{F}(U) \rightarrow \mathcal{F}(\{U_i \rightarrow U\})$  is fully faithful.
- (ii)  $\mathcal{F}$  is a *stack* over  $\mathcal{C}$  if for each covering  $\{U_i \rightarrow U\}$  in  $\mathcal{C}$ , the functor  $\mathcal{F}(U) \rightarrow \mathcal{F}(\{U_i \rightarrow U\})$  is an equivalence of categories.

Ex: the stack  $\mathcal{QCdh}^{\text{cart}}$

Ex: Stack of  $G$ -bundles

Ex: Quotient stacks

$\hookrightarrow$  these are algebraic,

To define this, some notions

- 1) base preserving functors
- 2) 2-fiber product

- 2) 2-fiber product
- 3) 2-yoneda lemma

2-YONEDA LEMMA. *The two functors above define an equivalence of categories*

$$\text{Hom}_{\mathcal{C}}((C/X), \mathcal{F}) \simeq \mathcal{F}(X).$$

- 4) definition of representable maps